① a continuous Random Variable x takes the value between $x = 2$ and $x = 5$ has a density function given by $f(x) = k(1+x)$. Find $P(X<4)$ and $P(3<x<4)$

Sol: $f(x) = k(1+x)$, $2 \le x \le 5$

To find k,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_2^5 k(1+x)\,dx = 1$$

$$\int_2^5 (k)+(x)\,dx = 1$$

$$k\int_2^5 (1+x)\,dx = 1$$

$$k\left[x + \frac{x^2}{2}\right]_2^5 = 1$$

$$k\left[5 + \frac{25}{2} - 2 - \frac{4}{2}\right] = 1$$

$$k\left[\frac{10+25}{2} - \frac{4-4}{2}\right] = 1$$

$$k\left[\frac{10+25-8}{2}\right] = 1$$

$$k\left[\frac{35-8}{2}\right] = 1$$

$$k\left[\frac{27}{2}\right] = 1$$

$$k = 2/27$$

$$f(x) = \frac{2}{27}(1+x), \quad 2 \le x \le 5$$

i) $P(x<4)$

$$= \int_{-\infty}^{4} f(x)\,dx$$

$$= \int_{-\infty}^{2} 0\,dx + \int_2^4 f(x)\,dx$$

$$= \int_2^4 \frac{2}{27}(1+x)\,dx$$

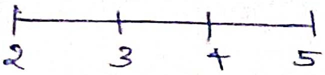
$$= \frac{2}{27}\int_2^4 (1+x)\,dx$$

$$= \frac{2}{27}\left[x + \frac{x^2}{2}\right]_2^4$$

$$= \frac{2}{27}\left[4 + \frac{16}{2} - 2 - \frac{4}{2}\right]$$

$$= \frac{2}{27}\left[\cancel{4} + 8 - \cancel{2} - \cancel{2}\right]$$

$$= \frac{16}{27}$$



$$= \int_{-\infty}^{2} 0\,dx + \int_{2}^{3} 0\,dx + \int_{3}^{4} f(x)\,dx + \int_{4}^{x} 0\,dx.$$

$$= \int_{3}^{4} 2/27 \;(1+x)\,dx$$

$$= \frac{2}{27} \int_{3}^{4} (1+x)\,dx.$$

$$= \frac{2}{27} \left[x + \frac{x^2}{2}\right]_3^4$$

$$= \frac{2}{27} \left(4 + \frac{16}{2} - 3 - \frac{9}{2}\right]$$

$$= \frac{2}{27} \left(4 + 8 - 3 - \frac{9}{2}\right]$$

$$= \frac{2}{27} \left(12 - 3 - \frac{9}{2}\right]$$

$$= \frac{2}{27} \left[9 - \frac{9}{2}\right]$$

$$= \frac{2}{27} \left[\frac{18-9}{2}\right]$$

$$= \frac{\cancel{2}}{27} \left[\frac{9}{\cancel{2}}\right]$$

$$= \frac{9}{27} = \frac{3}{9} = \frac{1}{3}.$$

$$2/27 \int_{3}^{4} (1+x)\,dx.$$

$$2/27 \left[x + \frac{x^2}{2}\right]_3^4$$

$$\frac{2}{27} \left[4 + \frac{\cancel{16}^{\,8}}{\cancel{2}_{\,1}} - 3 - \frac{9}{2}\right]$$

$$\frac{2}{27} \left[12 - \frac{3}{1} - \frac{9}{2}\right)$$

$$\frac{2}{27} \left[12 - \frac{6 - 9}{2}\right]$$

$$\frac{2}{27} \left[\frac{24 - 6 - 9}{2}\right]$$

$$\frac{\cancel{2}}{27} \left[\frac{24 - 15}{\cancel{2}}\right]$$

$$\frac{9}{27} = \frac{3}{9} = \frac{1}{3}.$$

P(

$p(x > 10)$ ...

3) The probability of a man hitting a target is 1/4

i) If he fires 7 times, what is the probability of hitting the target at least twice?

ii) How Many times must he fire so that the probability of hitting the target at least once is greater than 2/3?

Sol:

i) Exponential Distribution:

$$n = 7 \quad p = 1/4.$$

$$p + q = 1$$

$$\frac{1}{4} + q = 1.$$

$$q = 1 - \frac{1}{4}$$

$$q = 3/4.$$

p(atleast twice) =

$$p(x \geq 2) = 1 - p(x=0) - P(x=1)$$

$$p(x=r) = nC_r\, p^r q^{n-r}$$

$$= 1 - \left(7C_0 \left[\frac{1}{4}\right]^0 \left[\frac{3}{4}\right]^7 + 7C_1 \left[\frac{1}{4}\right]^1 \left[\frac{3}{4}\right]^6 \right)$$

$$= 1 - \left(1 \left[\frac{3}{4}\right]^7 + 7 \left[\frac{1}{4}\right] \left[\frac{3}{4}\right]^6 \right)$$

$$= 1 - \left(\frac{3^7}{4^7} + \frac{7 \times 3^6}{4^7} \right)$$

$$= 1 - \frac{3^6 [3+7]}{4^7} = \frac{1 - 3^6 (10)}{4 \times 4^6}$$

$$= 1 - \frac{\cancel{10}^{5} \times 3^6}{\cancel{4}_{2} \times 4^6} = 1 - \frac{5}{2}\left(\frac{3}{4}\right)^6 //$$

"/ P(atleat once great than 2/3).

$P(X = r) = nC_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{n-r}, \quad r = 0, 1, 2, \ldots$

$= P(X \geq 1) > 2/3$

$= 1 - P(X = 0) > 2/3$

$= 1 - nC_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^n > \frac{2}{3}$

$= 1 - \left(\frac{3}{4}\right)^n > \frac{2}{3}.$

$\left(\frac{3}{4}\right)^n < \frac{1}{3} \Rightarrow n = 4, 5, 6 \ldots$

Hence the man must fire atleast 4 times.

(4). A manufacturer of cotton pin knows that 5%. of his product is defective If he sells cotten pins in boxes of 100 and gaurantees that not more than 10 pins will be defective approximately probability that box will fail to meet granteed quality is.

sol: $P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$ (poisson distribution)

$n = 100 \qquad p = 5\% = \frac{5}{100}$

~~mean~~ :

$\lambda = np.$

$\lambda = 100 \times \frac{5}{100}$

$\lambda = 5.$

$P(X = x) = \frac{e^{-5} 5x}{x!}$

$P(X>10) + P(X \le 10) = 1.$

$$P(X>10) = 1 - P(X \le 10).$$

$$= 1 - \sum_{n=0}^{10} \frac{e^{-5x} \, 5x}{x!}$$

$$= 1 - \frac{e^{-5(10)} \, 5(10)}{10!}$$

$$= 1 - \frac{e^{-50} \, 50}{10!}$$

$$= 1 - \frac{(1.9287)(50)}{10!}$$

$$= 1 - \frac{(1.9287)(50)}{3628800}.$$

$$= 1 - 2.6574$$

$$= 1.6574.$$

5) Sol:

X is Uniformly distributed. on $(0, 30)$.

$$f(x) = \begin{cases} \frac{1}{30}, & 0 < x < 30 \\ 0, & \text{otherwise.} \end{cases}$$

$$P\{\text{wait atleast 20 min}\} = P[X > 20]$$

$$= \int_{20}^{30} f(x)\,dx. = \int_{20}^{30} \frac{1}{30}\,dx.$$

$$= \frac{1}{30}\left[30 - 20\right] = \frac{10}{30} = \frac{1}{3}$$

⑥. Sol.

Let Y denotes daily consumption of milk

$X = Y - 2000$

exponentially distributed. $\lambda = \dfrac{1}{3000}$.

$$f(x) = \lambda e^{-\cancel{(x/\lambda)}} \quad = \lambda e^{-\lambda x}.$$

$$= \frac{1}{3000} e^{-x/3000} \quad ; \; x > 0$$

P{stock insufficient for one day} =

$$P(Y > 35000) = P(X + 20000 > 35000).$$

$$P(X > 15000) = \frac{1}{3000}\int_{15000}^{\infty} e^{-x/3000}\, dx = e^{-5}$$

P(stock insufficient for two days $= (e^{-5})^2 = e^{-10}$

2) Let the Random Variable X have the pdf $f(x) = \begin{cases} \frac{x}{4} e^{-\left(\frac{x}{2}\right)} & ; x > 0 \\ 0 & ; \text{otherwise} \end{cases}$

Find the moment Generating function and the first four moments about Origin.

Sol:

To find MGF:

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$$

$$= \int_0^{\infty} e^{tx} \frac{x}{4} e^{-x/2}\, dx.$$

$$= \frac{1}{4} \int_0^{\infty} e^{tx}\, x\, e^{-x/2}\, dx.$$

$$= \frac{1}{4} \int_0^{\infty} e^{tx - x/2}\, x\, dx.$$

$$= \frac{1}{4} \int_0^{\infty} \underbrace{e^{(t - 1/2)x}}_{v.}\, \underbrace{x}_{u}\, dx.$$

$u = x \qquad V = e^{(t - 1/2)x}$

$u' = 1 \qquad V_1 = \dfrac{e^{(t-1/2)x}}{t - 1/2}.$

$u'' = 0. \qquad V_2 = \dfrac{e^{(t-1/2)x}}{(t - 1/2)^2}$

$$M_x(t) = \frac{1}{4}\left[\frac{x\, e^{(t - 1/2)x}}{\left(t - \frac{1}{2}\right)} - \frac{e^{(t - 1/2)x}}{(t - 1/2)^2}\right]_0^{\infty}$$

$$= \frac{1}{4} \cdot \left[\frac{x e^{(t-1/2)x}}{(t-1/2)} - \frac{e^{(t-1/2)x}}{(t-1/2)^2} \right]_0^\infty$$

$$= \frac{1}{4} \left(\frac{x e^{-(1/2-t)x}}{(t-1/2)} - \frac{e^{-(1/2-t)x}}{(t-1/2)^2} \right]_0^\infty$$

$$= \frac{1}{4} \left[(0-0) - \left[0 - \frac{1}{(t-1/2)^2} \right] \right]$$

$$= \frac{1}{4} \left[\frac{1}{(t-1/2)^2} \right]$$

$$M_x(t) = \frac{1}{4(t-1/2)^2} \rightarrow \text{MGF}.$$

) first four moments about origin

$$M_r' = E(x^r) = \int_{-\infty}^{\infty} x^r f(x)\, dx$$

$$= \int_0^{\infty} x^r \cdot \frac{x}{4} e^{-x/2}\, dx.$$

$$= \frac{1}{4} \int_0^{\infty} x^r x e^{-x/2}\, dx$$

$x = 2t$, $dx = 2\, dt$

$$= \frac{1}{4} \int_0^{\infty} 2t^r (2t)\, e^{-2t/2}\, 2 \cdot dt$$

$$= \frac{1}{2} \int_0^{\infty} 2t^r (2t)\, e^{-(2t/2)}\, dt$$

$$= \frac{1}{2} \int_0^\infty 2t^r (2t)\, e^{-2t/2}\, dt.$$

$$= \frac{1}{2} \int_0^\infty 4t^{r+1}\, e^{-t}\, dt.$$

$$= \frac{4}{2} \int_0^\infty t^{r+1}\, e^{-t}\, dt.$$

$$\Gamma n = \int_0^\infty e^{-x} x^{n-1}\, dx$$

$$= 2 \int_0^\infty t^{(r+2)-1}\, e^{-t}\, dt$$

$$= 2 \int_0^\infty e^{-t}\, t^{(r+2)-1}\, dt.$$

$$\Gamma n = (n-1)!$$

$$\boxed{M_r' = 2\,\Gamma r+2}$$

$$\Gamma n+1 = n!$$

To find moment about the origin

$\mu_r' = 2\,\overline{|(r+2)}$

$r=1 \quad \mu_1' = 2\,\overline{|3}$

$= 2(2)!$

$= 4$

$\boxed{\mu_1' = 4}$

$r=2 \quad \mu_2' = 2\,\overline{|4}$

$= 2(3!)$

$\boxed{\mu_2' = 12}$

$r=3 \quad \mu_3' = 2\,\overline{|5}$

$= 2(4!)$

$\boxed{\mu_3' = 48}$

$r=4 \quad \mu_4' = 2\,\overline{|6}$

$= 2(5!)$

$\boxed{\mu_4' = 240}$